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NETWORKS OF SPRINGS: A PRACTICAL APPROACH

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In the current work, we study the propagation of perturbations through networks of springs which are spatially distributed on a plane. We show that the topological properties of the network are related to the dissipation of energy within the system. By varying the *rewiring* parameter of the graph, and thus going from a regular to a random structure, we obtain a lower energy output, due to the fact that the initial (linear) perturbation is transformed into oscillations around each node. The results obtained are related to the transmission of information through a complex structure with potential applications to the design of more efficient damping systems.

Keywords: Complex networks; damping systems; long-range effects; information speed.

1. Introduction

The field of complex networks has attracted the attention of many researchers over the last years, due to the ease of applying concepts coming from complex networks theory to different real-world problems. In particular, many interesting results have been obtained in social networks, communication and traffic control, or system security and failure analysis [Boccaletti *et al.*, 2006; Newman, 2003]. Most of the real networks that have been analyzed are based on dimensionless graphs, where the spatial distribution of nodes is disregarded, either because it is not necessary or because it does not affect the structure or dynamics of the network. Nevertheless, the position of nodes may have a relevant role in different processes occurring in many networks, such as disease or rumor transmission, information flow or transport phenomena. In general, the study of spatial (complex) networks has not evolved as much as their adimensional counterparts, and few works have dealt with the structural topology of different spatial networks [Barrat *et al.*, 2005] such as the railway or subway network [Latora & Marchiori, 2002; Sen *et al.*, 2003], the network of streets inside a city [Cardillo *et al.*, 2006; Youn *et al.*, 2008] or the network of connections between airports [Zanin *et al.*, 2008]. In this work, we focus on the study of a particular spatial network with potential applications in engineering: a two-dimensional network of springs. We depart from a regular topology consisting of a set of springs connected to their first neighbors and we apply a rewiring probability p to the connections. Under these conditions, we study how energy (coming from an external perturbation) is transmitted through the network and, particularly, how certain rewiring ranges reduce the response of the system, or in other words, damp the effect of external perturbations.

2. The Physical System

Figure 1 shows a qualitative representation of the kind of network under study. Nodes represent masses (of unitary weight), which are connected to each other through classical springs that follow the differential equation:

$$-k_i l_i - \beta \frac{dl_i}{dt} = m \frac{d^2 l_i}{dt^2} \tag{1}$$

where l_i is the displacement of the spring i from its natural length, k_i is the force constant, m is the mass of the nodes and β is a certain damping coefficient. All parameters are equal for the N nodes of the network (see caption of Fig. 1 for details). All nodes are distributed over the XY plane, as well as the external perturbations, which restrict any movement to the horizontal plane (two-dimensional motion). Specifically, the external input consists of a displacement of the nodes of the bottom row of the network in a direction parallel to the Y axis. Finally, masses at the left and right boundaries are fixed. Under these conditions, different perturbations are applied at the base of the network (see arrows in Fig. 1), and we consider the uppermost row as the output of the system. In this way, we treat the spring network as a "black box", which responds with a certain output when an input perturbation is applied.

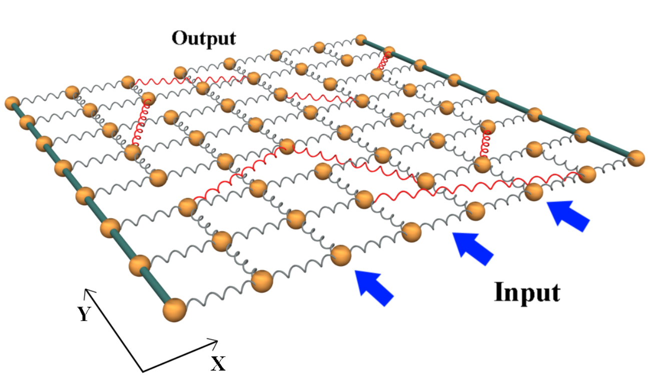

Fig. 1. 3D graphical representation of a two-dimensional 8×8 springs network. Lateral nodes are fixed and perturbations (blue arrows) are applied to the first row of the network ($y = 0$). The parameters used in the simulations are: $\kappa_i = \kappa = 1\,\text{N/m}$, $m_i = m = 1\,\text{kg}$ and $\beta_i = \beta = 1\,\text{Ns/m}$.

When links between nodes are arranged in a regular structure (i.e. each node is connected with its four nearest neighbors) the system behaves as a classical spring, i.e. propagates a damped wave with a well defined frequency. Nevertheless, when a *rewiring* probability p is introduced in the node connections (note certain rewirings in Fig. 1), and an external perturbation is applied to the bottom row, the propagation of energy towards the upper row suffers important nonlinear effects. The internal rewiring breaks the symmetry of the system and nodes oscillate around its equilibrium point in a nonperiodic motion. As a result, part of the energy propagates in the X direction, leading to a reduction of the amplitude of the oscillations in the last row (output) of the network. In the following section, we describe the effect of the rewiring probability on the energy propagation.

3. Numerical Results

One of the aims of this work is to find network structures that reduce the output oscillations of the system when a perturbation is applied, even if damping is not considered. In this way, we would have increased the efficiency of a damping system using only passive (non-dissipative) constructions.

Figure 2 (left) shows the maximum displacement at the output (uppermost row of the two-dimensional network) when a (fixed) vertical perturbation is applied to the bottom row, as function of the rewiring probability p. Numerical simulations are repeated 500 times for each rewiring p in order to have enough statistics. We can observe how the output amplitude decreases rapidly, even for low rewiring probabilities, thus leading to a network that better absorbs the external perturbation. The rewiring effectively reduces the output energy due to the fact that each node transforms the linear (initial) perturbation into oscillations around their steady point. This point can be observed in Fig. 2 (right), where we plot the averaged probability distribution function of the displacement of all nodes from their steady state, for a rewiring of $p = 0.5$.

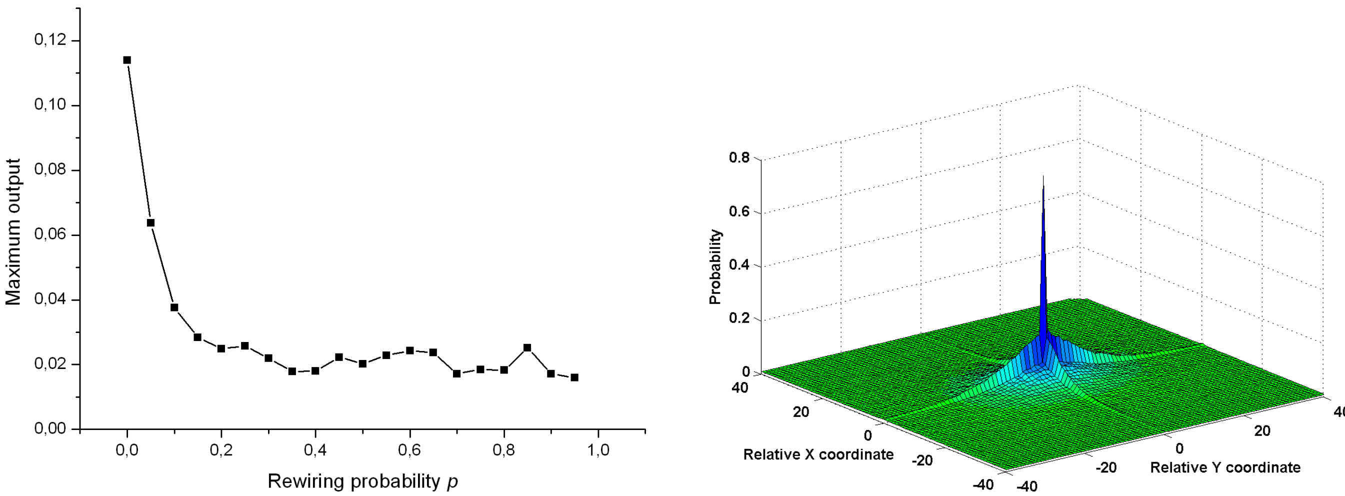

Fig. 2. (Left) Maximum output amplitude as a function of the rewiring parameter p. Thanks to the rewiring, the output amplitude is reduced. This is due to the appearance of oscillations around the equilibrium position of the nodes. (Right) Probability distribution (averaged over 10.000 repetitions) of the position of all nodes with respect to their equilibrium position $(0, 0)$ for $p = 0.5$.

The effect of the rewiring can be regarded as the sum of two different contributions:

(1) Since links (i.e. springs) can oscillate in any direction over the XY plane, and not only in the X or Y axis, the initial *down-up* movement is deflected and rotated in any direction, with a new X component that reduces the original displacement in the Y direction.
(2) Rewiring introduces long-range connections that break the symmetry and the synchronized motion of the system. In this way, when the main *front* of the wave is moving in the bottom-up direction, it can encounter other waves generated by long-range connections which travel in the opposite direction, thus creating destructive interferences.

In order to check the importance of the last point, i.e. the effect of long-range connections, we are going to study the influence of the mean link length in the output amplitude. With this aim, we construct a set of networks for each rewiring probability, ranging from $p = 0.1$ to $p = 1.0$. Next, we measure the mean link length for each network. Note that different networks with the same rewiring probability may have different mean lengths due to the stochastic nature of the rewiring. Finally, we measure the projection of the mean length in the Y axis, $\overline{L}$, since it is the direction of the initial perturbation. In this way, we are analyzing how the long range connections along the direction of the perturbation influence the propagation. In Fig. 3 (left) we plot the mean output displacement as a function of $\overline{L}$, for different rewiring probabilities. We observe that for high enough rewirings, there exists an optimum mean length, or in other words, for a given rewiring p, the networks that have the optimum mean length have the lowest amplitude at their output. For example, if we create a set of networks with probability $p = 0.5$, those networks with a mean length close to $\overline{L} = 2.25$ (see the minimum at Fig. 3) will damp the external perturbation more efficiently than others with the same p and different mean length. In Fig. 3 (right) the optimum value of $\overline{L}$ is plotted against the rewiring probability, along with the corresponding output amplitude.

3.1. *Periodic perturbations*

Spring-based systems have been widely studied in the past as a paradigmatic example of harmonic systems where phenomena such as resonance appear. The collapse of the old Tacoma Narrows bridge in Seattle is a well-known example of how a complex system responds to periodic perturbations [Tacoma Narrows Bridge, 1940] and how it can enter into a resonance regime with dramatic consequences to the system. With the aim of checking the oscillatory properties of the previous networks (regular and rewired), we change the external perturbation to a cosine force $F(t) = F_0 \cos(\omega t)$ which is applied

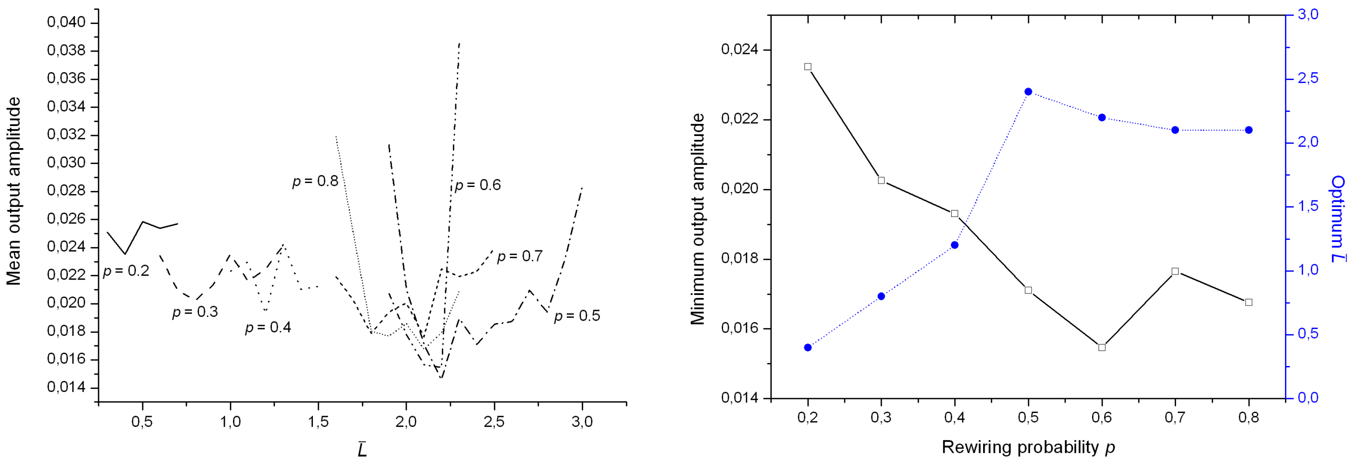

Fig. 3. (Left) Mean output amplitude as a function of $\overline{L}$, the projection of the mean length of the links in the direction of the perturbation. (Right) Minimum output amplitude (left vertical scale) and optimum $\overline{L}$ (right vertical scale) as a function of the rewiring probability p. Each rewiring probability p is repeated 2000 times with different initial conditions in order to have enough statistics.

to the lowest row of the network in the same direction as in the previous section. We also introduce a damping force ($\beta = 0.1$), allowing energy dissipation, in order to prevent the system from resonating to infinity.

Under these conditions we plot the maximum output displacement for different rewiring probabilities p (see Fig. 4). Interestingly, we observe the appearance of a resonance in the system, as indicated by the maximum in the displacement distribution. These results point out two interesting points. First, the higher the *rewiring probability* is, the lower the maximum output oscillation is. At the same time, higher rewiring probabilities lead to thicker bell-shaped distributions with smoother maxima. These results can be explained as a consequence of the long-range connections, that break the regular structure of the system and make the resonance more diffuse. Second, the resonant frequency is not a function of the *rewiring probability*. Rather the opposite, there exists a range of frequencies at which all networks resonate, no matter what the rewiring probability is.

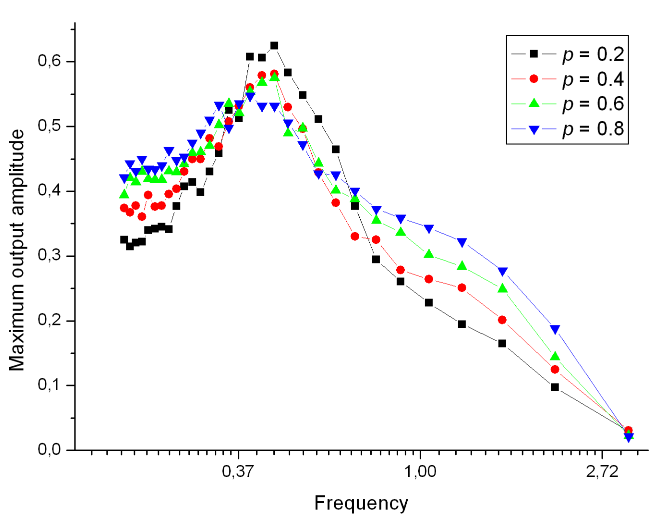

Fig. 4. Maximum output amplitude as a function of the frequency of the sinusoidal perturbation applied to the system, for different rewiring probabilities p.

4. A Practical Application: A Vehicle Damping System

From the numerical results exposed above, it is clear that the presence of a certain *rewiring* in the system can improve the performance of a regular damping mechanism. In mechanical engineering, for example, vehicle suspension design represents an important problem (see [Williams, 1997] for an exhaustive review of the field). The main aim of a suspension system is to isolate the vehicle from roadway irregularities and, at the same time, to reduce the roll movements while cornering, accelerating or braking. Nevertheless, in standard passive suspensions (i.e. not controlled electronically), the only parameter which can be easily tuned is the damping rate, and this requires the search of a difficult equilibrium: on one side, low damping reduces the discomfort experienced by occupants when the road presents discrete events, like

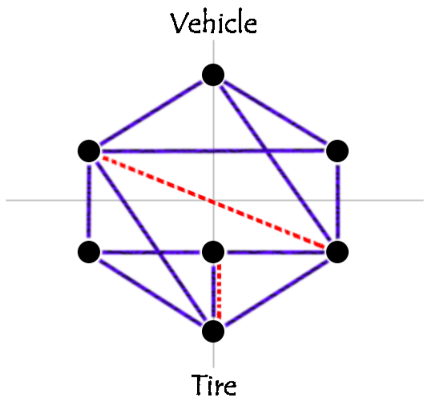

Fig. 5. Schematic representation of a damping system based on the application of a rewiring process to a regular springs/dampers (solid/dotted lines) structure.

steps or pot holes. This is because the step movement is filtered and smoothed by the damping system. For the same case, high damping would pass most of the acceleration to the vehicle, reducing the comfort of the car. On the other hand, each manoeuvre (like cornering, accelerating or braking) creates static loads on the tire and rolling movements that, in turn, produce interactions between the vertical and lateral dynamics of the vehicle. In this case, a high damping rate is preferable, as it reduces such oscillations and improves the overall dynamics.

The solution adopted by car industry is the inclusion of active or semi-active dampers, that change their characteristics according to the ride conditions. Nevertheless, those systems are expensive and include many components that increase the risk of failure.

What we propose is a passive system, built by several springs and dampers as displayed in Fig. 5. The configuration is created from a regular structure by rewiring some connections in order to break the symmetry.

In Fig. 6, we show an example of the efficiency of these kind of systems. On the vertical axis, we plot the Y component of the acceleration (of the uppermost node of the network) as a function of time for the rewired structure of Fig. 5 and an equivalent passive damping system. On the left plot, we consider a cornering manoeuvre, which is represented as a constant force acting on several time intervals. On the right plot, we apply a step input, i.e. a strong force acting during few time steps. In both cases, the overall acceleration is reduced in the rewired network and, at the same time, the rolling movement, i.e. the transient time before arriving to the steady state, is also shortened. From a practical point of view, such a system will have a better comfort level than low damping suspensions, but with the ride characteristics and response of higher damping systems.

Although these results are quite interesting for a real application, it is worth noting that the proposed system may raise different mechanical problems (such as dimension, weight, presence of horizontal forces, etc.) that cause difficulty for the implementation in a vehicle and that should be studied in detail from an engineering point of view.

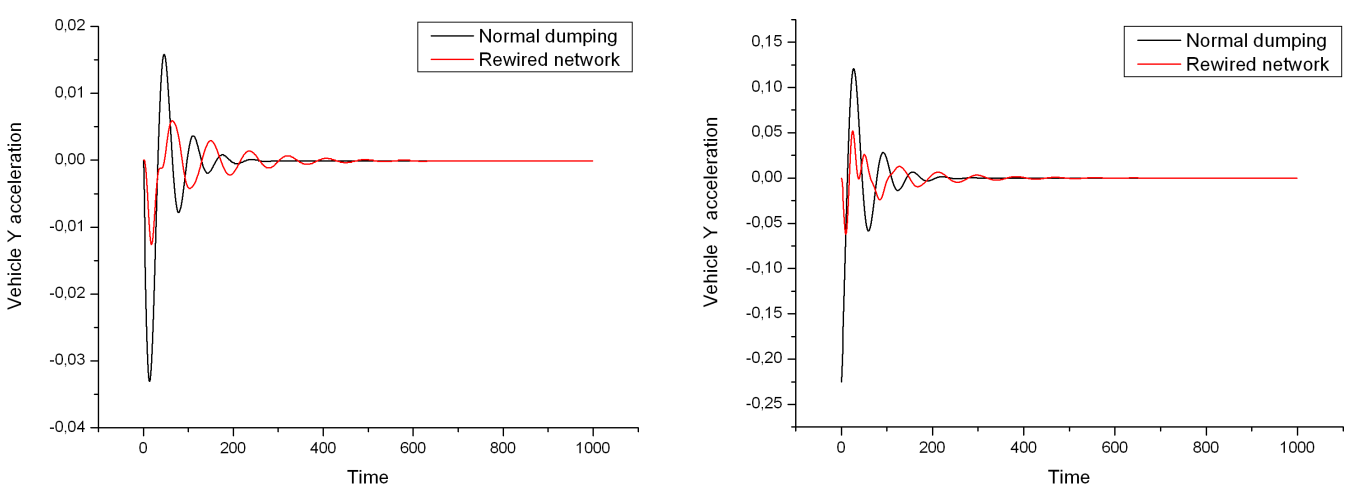

Fig. 6. Resulting Y (vertical) acceleration of the vehicle in a regular damping system and a (equivalent) rewired one, for a steering input (left) and a step input (right).

5. Conclusions

Despite complex networks being used as a tool for studying many real systems, less attention has been paid to spatial mechanical constructions. In this work, we have studied a two-dimensional network of springs, with special attention to the consequences of rewiring the network. We have shown that the symmetry breaking and the long range effects introduced by the rewiring of the internal connections lead to a reduction of the output amplitude of the system when external perturbations are applied. Furthermore, when periodic perturbations are considered, we observe the appearance of a resonance region, which does not depend on the rewiring parameter p. Finally, we have discussed the possible applications of this kind of systems to the design of efficient damping mechanisms, showing that they are an alternative to classical (passive) damping systems.

References

Barrat, A., Barthlemy, M. & Vespignani, A. [2005] "The effects of spatial constraints on the evolution of weighted complex networks," *J. Stat. Mech.* P05003.

Boccaletti, S., Latora, V., Moreno, Y., Chavez, M. & Hwang, D.-U. [2006] "Complex networks: Structure and dynamics," *Phys. Rep.* **424**, 175.

Cardillo, A., Scellato, S., Latora, V. & Porta, S. [2006] "Structural properties of planar graphs of urban street patterns," *Phys. Rev. E* **73**, 066107.

Latora, V. & Marchiori, M. [2002] "Is the Boston subway a small-world network?" *Physica A* **314**, 109–113.

Newman, M. E. J. [2003] "The structure and function of complex networks," *SIAM Rev.* **45**, 67–256.

Rosvall, M., Trusina, A., Minnhagen, P. & Sneppen, K. [2005] "Networks and cities: An information perspective," *Phys. Rev. Lett.* **94**, 028701.

Sen, P., Dagupta, S., Chatterjee, A., Sreeram, P. A., Mukherjee, G. & Manna, S. S. [2003] "Small-world properties of the Indian subway," *Phys. Rev. E* **67**, 036106.

Tacoma Narrow Bridge: There is a video of collapse of the Tacoma Narrows bridge (year 1940) available at: http://www.youtube.com/watch?v=HxTZ446tbzE.

Williams, R. A. [1997] "Automotive active suspensions," *Proc. Instn. Mech. Engrs. D* **211**, 415–444.

Youn, H., Gastner, M. T. & Jeong, H. [2008] "Price of anarchy in transportation networks: Efficiency and optimality control," *Phys. Rev. Lett.* **101**, 128701.

Zanin, M., Buldú, J. M., Cano, P. & Boccaletti, S. [2008] "Disorder and decision cost in spatial networks," *Chaos* **18**, 023103.